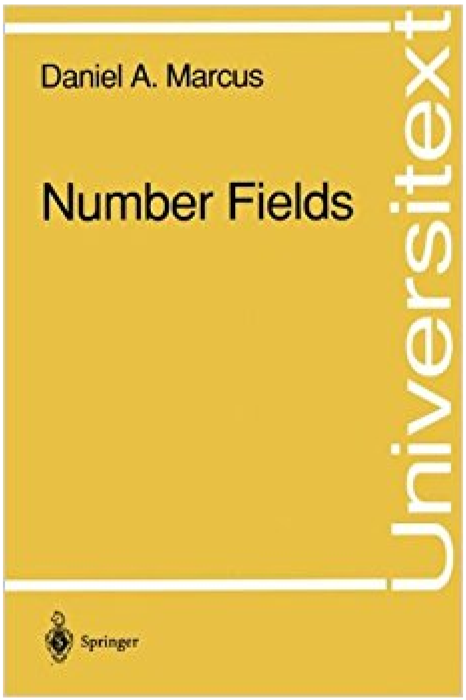
Number Fields (Universitext)



Synopsis

Requiring no more than a basic knowledge of abstract algebra, this text presents the mathematics of number fields in a straightforward, pedestrian manner. It therefore avoids local methods and presents proofs in a way that highlights the important parts of the arguments. Readers are assumed to be able to fill in the details, which in many places are left as exercises.

Book Information

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Customer Reviews

I got this book for a class in number theory. Although I can not find anything wrong with the book, I had a lot of trouble using it.Like I said in the title, the book is quite dense and if you do not already have a very firm understanding of abstract algebra some of the proofs can be hard to follow. This was not made any easier by the type font. (It really looked like it was typed on an old typewriter.) Also, in this day and age of LaTeX I am used to seeing mathematics with a host of strange symbols that cannot be found on any typewriter. In Number Fields there are only a dozen or so non-standard symbols. This may sound like a lot but when doing mathematics, trust me, it is not.I should point out that my low rating reflects the fact that I was using the book as a learning reference. If I were not a student seeing this material for the first time I most likely would have rated this higher. The problems at the end of each chapter were all well thought out thought provoking questions. (Sometimes this worked against us as the professor would be assigning problems and she would say "Number 10 is a good one so turn in that one. So is number 11 so do that one too. Wow! Number 12 is quite engaging...") Also, because it is so dense, the book manages to cover a lot of material in just a few pages. This makes the book very comprehensive.Overall if you are a young student, looking for a good reference on number fields, skip this book. On the other hand, if you are a more mature student of mathematics I would recommend this book as a way to deepen your understanding.

This book has a lot to offer by way of explanation and a solid development of content. Highly recommended. Great exercises.

This book was published, apparently, in 1977. I've learned of its existence in about 1985, in the undergraduate number theory seminar of the Mathematics Faculty of University of Bucharest. While it was very well regarded by students attending the seminar, I was absolutely thrilled with it. By then, the undergraduate two-year algebra course was close to being completed, hence all prerequisites were in place. The most advanced prerequisites were:- the structure theorem for finitely generated abelian groups- Sylow's Theorems (isolated cases)- basic complex-differentiable function theory (very little, could be taken on faith)- multivariable integration (very limited use)In an attempt to make the book virtually self-contained, three appendices present the needed material on commutative rings and their ideals, basic Galois theory for finite extensions of the rationals in the field of complex numbers, finite fields and rings, their Galois groups, respectively groups of units.The author goes very gingerly through the arguments throughout the book, with lots of details (suggested problems at the end of each chapter are often broken down into small, nearly obvious steps), and helpful hints (when it's about questions the reader is supposed to answer) are very frequent. He even explicitly states that he wanted to avoid the use of localization altogether, in order to make the text accessible to beginner math students! Cross references on proofs, statements, arguments strengthen the feeling of well-connectedness of this text.The book starts with a simple, charming case of Fermat's Last Theorem (though it had not been yet proved by Andrew Wiles), in the spirit of Kummer's work, as a motivation for the material to be treated. First, we learn (in chapter 2) the correct generalization of the ring of integers Z inside its field of fractions Q, in the case where Q is replaced by one of its finite extensions K: it's those elements of K which satisfy monic equations over Z. Here, a great deal of attention is given to determining the discriminant of R over Z, and the use of it in the determination of a canonical basis of R over Z. To the examples in the text (for quadratic and cyclotomic field extensions), many others in the problem section (treating biquadratic and some cubic extensions, as well as the real subextensions of the cyclotomic ones) are added. Moving up, the divisibility theory in Z is extended in chapter 3 to a divisibility theory for ideals in R, with a general prime factorization criterion (Dedekind's) given. The ramification criterion for a prime involving the discriminant is augmented in the problem section by the refined version which involves the different. Prime factorization, made once again exciting! Chapter 4 highlights the way Galois groups control prime factorization. The Quadratic Reciprocity Theorem receives an easy proof in this chapter. As a fantastic application, Weber's theorem (which states that every finite extension of Q with commutative Galois group lies inside a uniquely determined minimal cyclotomic extension) is proven as a set of accessible exercises. In chapter 5, the finiteness of the ideal class group of ideals of R is proven, with numerous examples of explicit determination (with more examples in the exercises). Dirichlet's theorem on the finite generation of the group of units of R is also proven here, and explicit determination of this group is made in a few important cases. It is worth noting that, as the book progresses, same (type of) field extensions of Q are revisited, in order to answer, in their case, the new questions under current study. So one ends up knowing quite a bit about certain finite field extensions of Q! Chapter 6 proves that ideals are uniformly distributed among classes of ideals. Concretely, the number of ideals in an arbitrary ideal class, of norm bounded by t>0, is asymptotically proportional to t, and the proportionality constant (independent of the chosen class) is determined. Further refinements of this are presented in the exercises. Chapter 7 offers an introduction into the study of the Dedekind Zeta Function (which generalizes Riemann's over Q, to K), with main application the determination of a class number (order of the ideal class group) formula in the case of quadratic fields. A few theorems on the distribution of primes are deduced in the exercises. Chapter 8 closes the book with a sketch of class field theory, while also deepening the study of prime distribution, including Tchebotarev's Density Theorem; Dirichlet's theorem on the infinitude of primes in an arithmetic progression is also given.While on a first reading, the solving of most problems may be skipped (making the reading relatively easy to a beginner student), going through the problems shows the true richness of the text, offering the reader an excellent introduction into the subject. Other books, which treat the subject with localizations, completions, valuations, will add further perspective on the subject.

As a graduate student learning about algebraic number theory this book has most of the core needed. Everything from dedekind domains, the class group, the splitting of primes, and the galois action on primes laying above in certain extensions. After talking with other students further into the subject, reading this book gave me a lot of "oooooooh" moments where you finally understand what someone said earlier.Also this book has a ton of exercises ranging from trivial to intense. Some are a little more computational, some so much so I decided it wasn't worth it. I would like a few more of the smaller theoretical property exercises. But the computations do help ground the work in something concrete which makes it easier to grasp at first. A lot of important concepts were introduced through exercises. Sometimes I wish there was a little more elaboration on some of these.Note that it is typewritten, as if on a typewriter. This text should get updated, it's very useful as a reading text. Maybe not so much as a reference.

Typewritten, easy to understand with good examples.

I have enjoyed reading this book. It's very down to earth and it presents the proofs as a series of exercises for the reader. It's a great way of being introduced to the finite extensions of rational numbers. The author shows how these fields can be very useful in solving the problems in elementary number theory, like prime factorizations of integers. Very engaging and enjoyable. A great introduction to the problems in algebraic number theory.--Alexander Shaumyan, poet and mathematician

I think the book is pretty well written for the ones with only exposure to basic abstract algebra.